Year 8 - Unit 4: Sequences

Learning Journey:

Prior Learning
- Perform addition, subtraction & multiplication accurately & recognise number patterns.
- Substitute values into simple algebraic expressions & understand the meaning of a variable.

Next Steps
- Solve more complex problems involving quadratic & non-linear sequences.
- Apply sequence skills to graphs, algebraic reasoning & GCSE problem-solving contexts.

By the end of this unit, you should be able to:
- Generate and describe arithmetic, geometric and Fibonacci sequences using term-to-term rules.
- Find and use the nth term of a linear sequence to calculate missing terms.

Keywords:

Sequence – A list of numbers that follows a pattern or rule.
Term – A single number in a sequence.
Arithmetic sequence – A sequence with the same difference each time.
Geometric sequence – A sequence found by multiplying by the same number each time.
Fibonacci sequence – A sequence where each term is the sum of the previous two terms.
nth term – A rule that gives any term in a sequence.

Calculating missing terms of a linear sequence

Take two consecutive terms from the sequence.
Subtract the first term from the next term to find the common difference d.
Add the common difference to the last term in the sequence to find the next term. Repeat for each new term.

Fill in the missing terms in the sequence 5, 8, ..., ..., 17.
$d = 8 - 5 = 3$
$8 + 3 = 11$
$17 - 3 = 14$

The missing terms are 11 and 14.

Calculate the next term of a geometric and Fibonacci sequence.

In a **geometric sequence**, you **multiply** each term by a **common ratio** to get to the next term. For example,

$0.5, \overset{\times 3}{\rightarrow} 1.5, \overset{\times 3}{\rightarrow} 4.5, \overset{\times 3}{\rightarrow} 13.5, \overset{\times 3}{\rightarrow} 40.5, \overset{\times 3}{\rightarrow} \ldots$

Therefore, to find the next two terms of this sequence we have to multiply the preceding term by 3, so $40.5 \times 3 = 121.5$ and $121.5 \times 3 = 364.5$

The Fibonacci Numbers
The first few terms of the Fibonacci sequence are:

$$1,1,2,3,5,8,13,21,\ldots$$

The rule is the previous 2 terms are added together in the sequence to get the next term.

Find the term-to-term rule of an arithmetic sequence

Sequence with a term-to-term rule of +3.
We add three to the first term to give the next term in the sequence and then repeat this to generate the sequence.

$$4, 7, 10, 13, 16, \ldots$$
$$\overset{+3}{\rightarrow}\ \overset{+3}{\rightarrow}\ \overset{+3}{\rightarrow}\ \overset{+3}{\rightarrow}$$

Can substitute into position to term rules

Some **sequences** have a **rule** called an n^{th} **term rule**, which tells you how to generate terms.
To find the **first** term in the **sequence**, substitute $n = 1$ into the **expression**. To find the **second** term in the **sequence**, substitute $n = 2$ into the **expression**. And so on.
Example: A **sequence** has the n^{th} term rule $2n + 1$. Find the **first 3 terms**.

First term: $n = 1$	$2n + 1 = 2 \times 1 + 1 = 3$
Second term: $n = 2$	$2n + 1 = 2 \times 2 + 1 = 5$
Third term: $n = 3$	$2n + 1 = 2 \times 3 + 1 = 7$

Finding the nth term

In order to find the nth term of a linear sequence:
Step 1: find the common difference for the sequence.
Step 2: multiply the values for n = 1, 2, 3, ... by the common difference.
Step 3: add or subtract a number to obtain the sequence given in the question.

Find the n^{th} term for the sequence 5, 9, 13, 17, 21, ...

The common difference d = 4. So our sequence is the 4n sequence
Here, we generate the sequence 4n=4, 8, 12, 16, 20, ... (the 4 times table).

How do we get from 4 to 5, from 8 to 9, 12 to 13. We add one to each term so our sequence is 4n + 1.